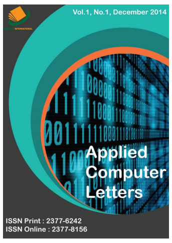
ZIBELINE INTERNATIONAL PUBLISHING

Applied Computer Letters (ACL)
DOI : http://doi.org/10.26480/acl.01.2017.14.16



THE APPLICATION OF UNILATERAL SINGLE VALUE CONTROL CHART BASED ON LOGNORMAL DISTRIBUTION

Shibo Xin, Xi Wang, Yanquan Chen

Department of Statistics, Beijing Technology and Business University, Beijing 102488, China
****Corresponding Author's E-mail: xsblm@163.com***

ARTICLE DETAILS

Article History:

Received 7 November 2017
Accepted 10 December 2017
Available online 5 January 2018

ABSTRACT

The risk theory tells us that the short-term high risks exist at the right tail of the distribution. So we advocate the design and parameter estimation method, which the special figures that reflect short-term risks are within the control limits of unilateral single value control chart based on lognormal distribution, and we also put forward how to use unilateral single value control chart through examples. This essay aims to monitor the short-term risks by means of control chart and reduce unnecessary losses of high risks.

KEYWORDS

Risk control, lognormal distribution, unilateral single value, control chart

I. INTRODUCTION

Based on a study, short-term risk assessment is one of the important parts of risk theory [1]. Present work mostly establish short- term risk models to describe the fluctuation process of the risk, while the research on the short-term risk fluctuation monitor is rarely seen [2]. According to research, the control chart method in the quality management is an important method to monitor the short-term risk [3]. The normal distribution which special figures reflect the short-term risks mostly shows right-side character (for example, lognormal distribution). High risk is right-skewed. Recent researches prove that the risk fluctuation monitor of the right-skewed distribution has its theoretical and practical significance[4-7]. Through measurement system analysis, identification of auto-correlation, normality test and Data Collection, we can get how to use unilateral single value control chart and avoid wrong analyses in quality control [8]. We can also use Statistical Process Control to monitor the result of the measurement. Statistical process control originated in the manufacturing because of its warning function. It can inform people of the potential risks promptly to make people make preparations [9].

According to a research, control chart is one of the seven quality engineer tools [10]. Since a researcher put forward normal control chart in 1924 to 1980s, the research on the method of control chart has made some progress [11]. The researcher's control chart takes a very important part in improving quality and productivity since included in the international standard [12]. Then, there are various kinds of control charts gradually. For example, The exponentially weighted moving average (EWMA) control chart are widely used in industries for monitoring small and moderate process shifts. The sum of squares generally weighted moving average (SS-GWMA) control chart to simultaneously detect both the increase and decrease in the variability [13]. In the international standards ISO8258: 1991 and GB / T4091-2001. Metering control chart what is assumed to quality characteristics follows a normal distribution. As for the problem of monitoring during the process of mass production, control chart contributes to improving the quality of products. However, putting SPC (Statistics Process Control) into use for short-term risk assessment has great limitations because mostly the random variable reflecting the short- term risk obeys right-skewed distribution [14].While the normal distribution N(μ, σ 2) is symmetry. And the high risks concentrate in the right of the distribution, so making control chart to monitor risks just needs making UCL (Upper Control Limit).For this reason, an appropriate design is necessary before it is used. This essay is intended to research unilateral single value control chart based on lognormal distribution and give analysis of example.

Based on a research, control chart is one of the statistical design charts that evaluate, record and assess the process quality control [15]. And it is graphical verification of assumption. The identification of various unnatural patterns that usually are exhibited in quality control charts leads to more focused diagnosis. And there are two kinds of errors in the verification of assumptions: one is α (abandon the right) and the other is β (get the wrong). In the theory of control chart, α is accidental and unavoidable while β is non- accidental and avoidable. We can detect the abnormal factors,

Therefore, on the basis of existing research this paper established the model of two generations product diffusion considering the unit cost with learning effects, and obtained the profit expression of innovative products, then analyzed the diffusion behavior of two generations' products based on the analysis of the process of product diffusion factors by using MATLAB simulation. Using dynamic simulation method to build the model to analyze the influence of each factor, this paper provided necessary basis for the decision-making for the enterprise market. promptly warn and reduce loses by means of the control chart. In short-term risks control, α is the probability that actual risk loses should be claimed but not, while β is the probability that actual risk loses doesn't happen but claimed. These two errors will both cause losses to the insurance company, but the change direction of α is opposite to β. As a result during making the control chart we should submit the principle that α plus β is the least. In this chapter we still use 3σ rules, α=0.0027.

2. UNILATERAL SINGLE VALUE CONTROL CHART

Suppose the special figure reflecting the short-term risk, X obeys lognormal distribution LogN (μ, σ^2) According to probability theory the density function of lognormal distribution is

$$f(x;\mu,\sigma^2)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\sigma}\dfrac{1}{x}e^{-\frac{1}{2}(\frac{\ln x-\mu}{\sigma})^2}, & x\ge 0\\ 0, & x<0\end{cases} \tag{1}$$

$f(\bullet)$ is the density function of lognormal distribution, $\mu\sigma^2$ are parameters and $\ln(X) \sim N(\mu, \sigma^2)$. Given α, X_α is upper α quantile in $LogN(\mu, \sigma^2)$, and $P(X \le X_\alpha) = 1-\alpha$. According to Shewhart control chart, bilateral single value control chart $Y \sim N(\mu, \sigma^2)$,

$$UCL=\mu+Z_{\alpha/2}\sigma \quad CL=\mu \quad LCL=\mu-Z_{\alpha/2}\sigma \quad (2)$$

UCL is the upper control limit CL is control center line, LCL is Lower control limit, Z_α is upper α quantile in $N(0, 1)$. From (2) we can see $UCL=\mu+Z_\alpha\sigma$ actually is Y's $\alpha/2$ quantile, $LCL=\mu-Z_{\alpha/2}\sigma$ is Y's $1-\alpha/2$ quantile, $CL=\mu$ is Y's mathematical expectation $E(Y)$. So the control limit of $X \sim LogN(\mu, \sigma^2)$'s unilateral single value control chart is

$$UCL=X_\alpha \quad CL=E(X) \quad (3)$$

$E(X) = e^{\mu+\sigma^2/2}$, just find the expression of X_α. By $P(X \le X\alpha) = 1-\alpha$, we get $P(X \le X_\alpha) = 1-\alpha$, we get $P(\ln(X) \le \ln(X_\alpha)) = 1-\alpha$, and $\ln(X) \sim N(\mu, \sigma^2)$, $\ln(X\alpha)$ is the quantile of normal distribution $N(\mu,\sigma^2)$. Coordinate transformation $Z\alpha = (\ln(X\alpha) - \mu)/\sigma$, given $\alpha = 0.0027$, from the standard normal distribution able, we know that $Z_{0.0027} = 2.78$, so $X_\alpha = e^{2.78\sigma+\mu}$, we get

$$UCL=e^{2.78\sigma+\mu} \qquad CL=e^{\mu+\sigma^2/2} \qquad (4)$$

When μ, σ is known, (2) is $X \sim LogN(\mu, \sigma^2)$ unilateral single value control chart (α = 0.0027). When μ, σ is unknown, it comes below.

Suppose there are many loss samples $X_1, X_2, \cdots, X_n$, and $X_i \sim LogN(\mu, \sigma^2)$, $i = 1, 2, \cdots, n$. Then take the log of $X_1, X_2, \ldots, X_n$, we get the special distribution distribution like this $\ln(X_1), \ln(X_2), \ldots, \ln(X_n)$. $\ln(X) \sim N(\mu, \sigma^2)$. Therefore, when,

$$\overline{\ln(X)} = \frac{1}{n}\sum_{i=1}^{n}\ln(X_i),$$

$$\rho_i = X_{(i)} / X_{(i+1)},$$

$$\ln(\rho_j) = |\ln(X_i) - \ln(X_{i+1})|,$$

$$\overline{\ln(\rho)} = \frac{1}{n-1}\sum_{j=1}^{n-1}\ln(\rho_j), \quad j = 1, 2, \cdots, n-1,$$

According to Shewhart's one-value range control chart parameters estimation method, we get

$$\hat{\mu} = \overline{\ln(X)} \qquad \hat{\sigma} = \frac{\sqrt{\pi}}{2}\overline{\ln\rho} \qquad (5)$$

We can prove $E(\hat{\mu}) = \mu$, $E(\sigma) = \hat{\sigma}$, $\overline{\ln(X)}$ is an unbiased estimation of μ. $\sqrt{\pi}\,\overline{\ln(\rho)}/2$ is unbiased estimation of σ. Put (5) into (4), we get the log normal distribution's unilateral single value control chart when μ, σ is unknown.

3. APPLICATION

During an insurance company inspection period, there are 30 policies claimed. The short-term risk indivi-dual claim is Table 1, and through former experience, individual claim obeys lognormal distribution. For example unilateral single value control chart application of step is below,

Table 1: Individual claim (Unit：Ten thousand yuan)

Num	Claim amount	Num	Claim amount	Num	Claim amount
1	30.55	11	30.36	21	14.97
2	8.94	12	11.70	22	13.75
3	10.35	13	17.94	23	5.96
4	4.42	14	25.06	24	21.21
5	10.27	15	21.23	25	4.46
6	5.42	16	1.97	26	9.51
7	16.42	17	40.93	27	14.71
8	19.59	18	24.08	28	15.98
9	1.71	19	4.03	29	7.01
10	4.95	20	3.49	30	17.46

Step 1: Calculate $\overline{\ln(X)} = 2.09$, $\overline{\ln(\rho)} = 0.86$. Estimate parameters $\hat{\mu} = 2.09$, $\hat{\sigma} = 0.76$;
Step2: Calculate control limit from (2.4)UCL = 66.87, CL = 10.79;
Step 3: Draw control line, trace points, then we get control chart of analysis which is shown in Figure 1;
Step 4: Judge whether the process of monitoring is steady through the analysis control chart drawn from step3. All sample points in Figure 1 are within the control limit and exist no non-random phenomenon,so we suggest that the process is under control;
Step 5: Prolong the control limit of control chart in stpe3 and finally we get the control chart of control.

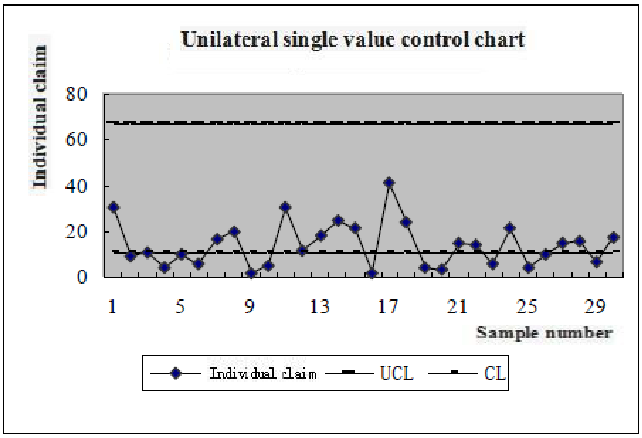


Figure 1: Unilateral single value control chart.

4. CONCLUSIONS

Control chart is an efficient tool to control risks. With the quality characteristic value in the lognormal distribution, the unilateral single value control chart given in the context, which reflects the short-term risks, is designed for the right distribution that high short-term risk exists. By monitoring the short-term risks, if charts appear the exception during the monitoring, it means the policy may have the risks of abnormal settlement of claims. Once this happens, you should find the causes (changes of the assurers' risks features or ethical risks etc.) ahead of time and give adjustments to cut down the losses in the future.

Comparing with other risk management devices, control chart is easy to operate. The operator can begin to work after simple training. Also, it can prevent the risk because the monitoring is for the process of the fluctuation of the risk.

ACKNOWLEDGEMENT

Finally we extend our sincere appreciation to Beijing Talents Funded Projects (Serial number: 2011D005003000011), Talents Teach-Youth Fellowship Program (Serial number: YETP1458) and Beijing Social Science Foundation Research Base Project (Project name: Beijing vegetable price volatility risk early warning system)

REFERENCES

[1] Xie, Z., Han, T. 2000. The Theory of Risk and Non-Life, Actuarial Mathematics, 75-80.

[2] Sheng, J., Xie, S., Pan, C. 2001. Probability Theory and Mathematical Statistics, Beijing: Higher Education Press, 83-90.

[3] Sun, G., Sun, J. 2003. New Edition of Quality Management, Second Edition, Beijing: Higher Education Press, 75-128.

[4] ISO 8258. 1991. Shewhart Control Charts.

[5] GB/T 4091. 2001. Shewhart Control Charts, Beijing: Standards Press of China, July.

[6] Jin, Y. 2014. Equipment Manufacturing Technology, China Metrology.

[7] Yu, J. 2012. Gaussian mixture models-based control chart pattern recognition, International Journal of Production Research, 50(23), 6746-6762.

[8] Lee, M., Michael, B.C. 2014. The synthetic mean square error control chart, Communications in Statistics, 43 (6),1523-1542.

[9] Lee, M.H. 2013. Variable sampling rate multivariate exponentially weighted moving average control chart with double warning lines, Quality Technology and Quantitative Management, 10(3),353-368.

[10] Alfaro, J., Ortega, Juan, F. 2013. A new control chart in contaminated data of t-Student distribution for individual observations, Applied Stochastic Models in Business and Industry, 29(1), 79-91.

[11] Shamsuzzaman, M., Zhang, W. 2012. Design of EWMA control chart for minimizing the proportion of defective units, International Journal of Quality & Reliability Management, 29(8), 836-850.

[12] Chen, Y.K. 2013. An evolutionary economic-statistical design for VSI X control charts under non-normality, The International Journal of Advanced Manufacturing Technology, 22(7-8), 602-610.

[13] Huang, C. 2014. A Sum of Squares Generally Weighted Moving Average Control Chart, Communications in Statistics – Theory and Methods 43(23), 5052-5071.

[14] Gauri, S.K., and S. Chakraborty, S. 2007. A study on the various features for effective control chart pattern recognition, The International Journal of Advanced Manufacturing Technology 34(3-4), 385-398.

[15] Celano, G., Castagliola, P., Trovato, E., Fichera, S. 2012. The economic performance of the Shewhart t chart, Quality and Reliability 28(2), 159-180.

Cite the article: Shibo Xin, Xi Wang, Yanquan Chen (2017). The Application of Unilateral Single Value Control Chart Based On Lognormal Distribution. *Applied Computer Letters,* 1(1) : 14-16.