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# DYNAMIC RELIABILITY OF TORPEDO STRUCTURE UNDER THE ARBITRARY RANDOM EXCITATION

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**ABSTRACT**

Aimed at the correlation between the impact loads and planing forces of the tail on supercavitating torpedo structure, the pseudo excitation had been constructed about every eigenpair of power spectral density matrix. By using Newmark method with pseudo excitation perturbation method, the computing equation of dynamic reliability are educed from the first excursion probability mechanism. Finally, the method of this paper was proved feasible comparing with the Monte Carlo method by an example.

**KEYWORDS**

Torpedo, reliability, random excitation, pseudo excitation

## 1. INTRODUCTION

Supercavity technology makes most of the surface of the underwater vehicle surrounded by low-density water vapor or gas without direct contacting with water, only the head of cavitator and the tail small part surface contact with water. This makes the vehicle achieve ultra-high-speed underwater navigation. According to a study, there are more researches at home and abroad on the fluid properties of supercavity, while less on the structural dynamic reliability of high-speed underwater supercavity vehicle [1-4]. As for the underwater vehicle whose speed is faster than 300 m·s -1, because of its high speed, so that any small perturbation will lead to violent collision between the tail and cavity wall of vehicle, thereby the deformation and destruction of vehicle is prone to take place due to high frequency vibration, Therefore, there is an important significance for researching structural dynamic reliability on supercavity vehicle under impact loads.

So far People have done a large number of theoretical analysis and experimental research on the vibration characteristic of vehicle structure, and obtained some important achievements. However, the randomness of physical and geometrical parameters of the structure is almost not considered in these studies. But in practice, the randomness of material properties and the variety of random factors in manufacturing and the installation process will result in randomness of structural dynamic characteristics. Based on the structural dynamic reliability method, and in view of the randomness of modulus of elasticity and density, the paper studied the dynamic reliability of torpedo with stochastic parameters under arbitrary random excitation.

## 2. FORCE ANALYSIS OF STRUCTURE

The torpedo will be unstable and collide with cavity walls when the torpedo is disturbed by a tiny disturbance. Based on a study, the forces of torpedo structure subject to as shown in Figure 1. $F_n$ is fluid resistance, $F_{pr}$ is axial thrust on the tail, $F_c$ and $F_D$ are impact load and sliding force, the calculation formula refer to literature [5-6].

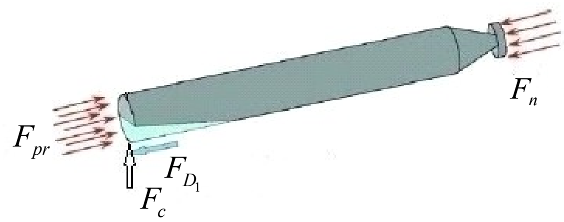


**Figure1:** Sketch of a torpedo subject to loads.

## 3.RANDOM RESPOND OF STRUCTURE WITH STOCHASTIC PARAMETERS

According to research, when the torpedo structure of the linear stochastic parameters is under polygene stationary random excitation, the equation of motion is[7].

$$Mu + Cu + Ku = R_f\{ f (t)\} \tag{1}$$

Where, M , C, K is respectively n th mass, damping and stiffness matrix, suppose that they all do not change with time. They are decomposed into the sum of average component and the random component of zero-mean, that is of average component and the random component of zero-mean, that is

$$M = M + M, C = C + C, K = K + K \tag{2}$$

Where, Rf is an invariable vector, which expresses the status of outside force distributing; supposing that the power spectral density function is given. If we do not consider the randomness of mass, damping and stiffness matrix, substitute their mean value into Equation (1), we can get

$$Mu + Cu + Ku = R_f\{f(t)\} \tag{3}$$

Because of the process of the upper and the lower surface of the tail with the same random excitations, by virtue of the phase difference is existent, so it is very important to take account of the wave passage effect. The sliding resistances are completely related, and the impact load and sliding resistance are not relevant, the fluid resistance can be considered to be constant. The workload of calculating this type problem is right smart by using conventional method of stochastic vibration, we can consider this typological problem for generalized single excitation, and we can solve this problem simply by using pseudo-excitation method. The excitation spectrum density matrix can be written as the following form

$$\left[S_{ff}(\omega)\right]=\{f_1\}^*\{f_1\}^T+\{f_2\}^*\{f_2\}^T \quad (4)$$

Where

$$\{f_1\}=\begin{Bmatrix}1\\-e^{-i\omega\Delta\tau}\\0\\0\end{Bmatrix}\sqrt{S_{F_C}(\omega)}e^{i\omega t},$$

$$\{f_2\}=\begin{Bmatrix}0\\0\\1\\e^{-i\omega\Delta\tau}\end{Bmatrix}\sqrt{S_{F_D}(\omega)}e^{i\omega t},$$

Where $\Delta\tau$ is time interval of tail collision； $S_F$ $S_F$ are spectrum density of loads. Because of the power spectrum matrix is the Hermitian matrix, so the excitation spectrum matrix of the torpedo subject to is

$$\left[S_{ff}(\omega)\right]=\sum_{j=1}^{m}\lambda_j\{\psi_j\}^*\{\psi_j\}^T$$

Where express conjugate； T express matrix transpose; $\lambda_j$ and $\psi$ are Hermitian matrix eigenvalue and eigenvector, so

$$\left[S_{ff}\right]\{\psi_j\}=\lambda_j\{\psi_j\} \quad (7)$$

$$\{\psi_i\}\{\psi_j\}^T=\delta_{ij}=\begin{cases}1(i=j)\\ \end{cases} \quad (8)$$

So we construct pseudo excitation as follows

$$\{\tilde f\}=\psi^*\sqrt{\lambda_j}e^{i\omega t} \quad (9)$$

So $S_{ff}(\omega)$ can be written in the form below

$$\left[S_{ff}(\omega)\right]=\sum_{j=1}^{m}\{\tilde f_j\}^*\{\tilde f_j\}^T \quad (10)$$

Substitute Equation (9) and $F_n$ into Equation (3), can get the displacement response u(i). According to the relationship of stress and displacement

$$\tilde\sigma^{(i)}=DB\tilde u^{(i)} \quad (11)$$

So the power spectrum density of the corresponding stationary random stress response is

$$S_{\sigma\sigma}(\omega_i)=\tilde\sigma^{(i)*}\tilde\sigma^{(i)T}$$

Where, $\sigma^{(i)*\sim}$ and $\sigma^{(i)T}$ are respectively conjugate matrix and transposed matrix of the corresponding pseudo stress response when the frequency is the i th discrete point of frequency. So the mean of stationary random stress response, and the mean of variance of its derivative are

$$\bar\nu_\sigma^2=2\int_0^\infty S_{\sigma\sigma}(\omega)d\omega\approx 2\sum_{i=0}^{N}S_{\sigma\sigma}(\omega_i)\Delta\omega_i$$

$$\bar\nu_{\dot\sigma}^2=2\int_0^\infty \omega^2 S_{\sigma\sigma}(\omega)d\omega\approx 2\sum_{i=0}^{N}\omega_i^2 S_{\sigma\sigma}(\omega_i)\Delta\omega_i$$

Aimed at the influence of the parameters randomness on $\nu^2$ and $\nu^2$, it is assumed that the structural randomness is caused by a series of normal random variations $X=(X_1,X_2,X_n)$ ‥that don't change with time. Where each random variation is expressed as

$$X_i=\bar X_i+X_i \quad (15)$$

The structural randomness leads to The structural randomness o leads to response $\nu_0$ and $\nu_\sigma$ having randomness. Weσ disintegrate $\nu_\sigma$ and $\nu_\sigma$ in the same way, that is

$$\nu_\sigma=\nu+\nu_\sigma,\ \nu_{\dot\sigma}=\nu+\nu_{\dot\sigma} \quad (16)$$

We process $\nu^2$ and $\nu^2$ at the mean value point $X=(X_1, X_2, X_n)^T$‥ of the basic random variation by using Taylor expansion. And take the mean values on both sides at the same time, we can obtain

$$E[\nu_\sigma^2]=\bar\nu_\sigma^2+\frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\nu_\sigma^2}{\partial X_i\partial X_j}\mathrm{Cov}(\tilde X_i,\tilde X_j)\Big|_{X=\bar X}+\cdots \quad (17)$$

$$E[\nu_{\dot\sigma}^2]=\bar\nu_{\dot\sigma}^2+\frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\nu_{\dot\sigma}^2}{\partial X_i\partial X_j}\mathrm{Cov}(\tilde X_i,\tilde X_j)\Big|_{X=\bar X}+\cdots \quad (18)$$

Relative to the average component, each random component can be seemed as small component, if we just consider the first order perturbation, the variance of $\nu^2$ and $\nu^2$ can be calculated as the following equation

$$\sigma_{\nu_\sigma^2}^2=\mathrm{cov}(\nu_{\sigma k}^2,\nu_{\sigma l}^2)=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\nu_{\sigma k}^2}{\partial X_i}\Big|_{\bar X}\frac{\partial\nu_{\sigma l}^2}{\partial X_j}\Big|_{\bar X}Cov(X_i,X_j) \quad (19)$$

$$\sigma_{\nu_{\dot\sigma}^2}^2=\mathrm{cov}(\nu_{\dot\sigma k}^2,\nu_{\dot\sigma l}^2)=\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\nu_{\dot\sigma k}^2}{\partial X_i}\Big|_{\bar X}\frac{\partial\nu_{\dot\sigma l}^2}{\partial X_j}\Big|_{\bar X}Cov(X_i,X_j) \quad (20)$$

Where $Cov(X_i,X_j)=\rho_{ij}\sigma_{X_i}\sigma_{X_j}$, $\sigma_{X_i}$ and $\sigma_{X_j}$ are respectively the standard deviation of $X_i$ and $X_j$, $\rho_{ij}$ is the correlation coefficient. By using Newmark method and pseudo excitation perturbation method, we can solve the dynamic equation of structure, the stress and the derivative of it to parameters can be obtained.

## 4. DYNAMIC RELIABILITY ANALYSIS OF STRUCTURE

Based on a study, with regard to the torpedo structure in this paper, by virtue of the randomness of its physical parameter and geometric parameter, this will lead to structural dynamic reliability also having randomness [8-10]. By using moment method of solving numerical features of stochastic variable, we can obtain the mean value and variances of structural dynamic reliability of torpedo with stochastic parameter are respectively as following

$$\mu_R=\exp\{-\frac{1}{2\pi}\frac{\mu_{\nu_{\dot\sigma}}}{\mu_{\nu_\sigma}}T[\exp(-\frac{\lambda_u^2}{2\mu_{\nu_\sigma^2}})+\exp(-\frac{\lambda_d^2}{2\mu_{\nu_\sigma^2}})]\} \quad (21)$$

$$\sigma_R^2 = \mu_R^2\{\frac{1}{2\pi}\frac{\mu_{v_\sigma}}{\mu_{v_\sigma}^2}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})]+\frac{1}{2\pi}\frac{\mu_{v_\sigma}}{\mu_{v_\sigma}^4}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})\lambda_u^2+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})\lambda_d^2]\}^2\sigma_{v_\sigma}^2+\mu_R^2\{-\frac{1}{2\pi}\frac{1}{\mu_{v_\sigma}}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})]\}^2\sigma_{v_\sigma}^2+\mu_R^2\{\frac{1}{2\pi}\frac{\mu_{v_\sigma}}{\mu_{v_\sigma}^2}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})]+\frac{1}{2\pi}\frac{\mu_{v_\sigma}}{\mu_{v_\sigma}^4}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})\lambda_u^2+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})\lambda_d^2]\}\{-\frac{1}{2\pi}\frac{1}{\mu_{v_\sigma}}T[\exp(-\frac{\lambda_u^2}{2\mu_{v_\sigma}^2})+\exp(-\frac{\lambda_d^2}{2\mu_{v_\sigma}^2})]\}\sigma_{v_\sigma}\sigma_{v_\sigma} \quad (22)$$

## 5. RESULT AND CONCLUSIONS

Some parameters of the examples in this paper refer to reference. In order to verify the method in this paper, we only consider several cases that either of parameters ( E , ρ ) is stochastic variable or both of them are stochastic variables at the same time. Analyze and calculate dynamic reliability of the maximum stress response element of the torpedo structure when coefficient of variation of parameters ( E , ρ ) are different values. Figure 2 shows the mean value curve of dynamic reliability of the torpedo structure .where “all” means all the coefficient of variation have the same values. From which we can see that the variability of modulus of elasticity has larger influence on dynamic reliability while the variability of density of mass has smaller influence on it. With time increasing, the influence of the variability of random parameter on dynamic reliability will also become larger. Figure 3 shows the mean value curve of dynamic reliability when the coefficients of variation of all the random parameters change at the same time. From which we can see that with the coefficients of variation increasing, the mean values of dynamic reliability diminish. That is to say with coefficient of variation of random parameters increasing, the influence on the mean value of dynamic reliability of torpedo structure will become larger

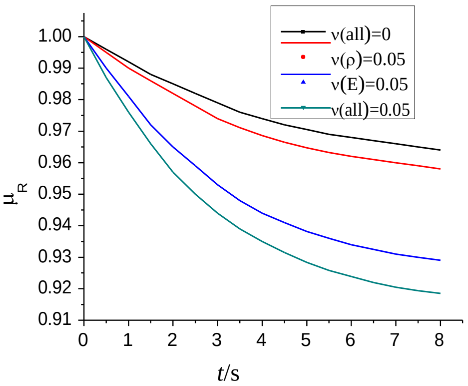


**Figure 2:** The curve of $\mu_R$ under different environment.

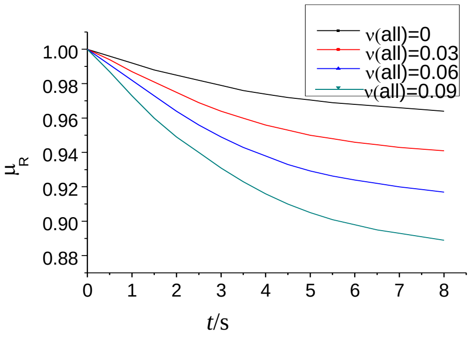


**Figure 3:** The curve of $\mu_R$ under different variances.

The pseudo excitation had been constructed about every eigenpair of power spectral density matrix in this paper. By Newmark method with pseudo excitation perturbation method, the computing equation of dynamic reliability are educed from the first excursion probability mechanism. The influence of the variation of random parameter on dynamic reliability of torpedo structure was given in this paper. We can obtain the following conclusions through the examples and analysis results.

The variation of modulus of elasticity E has a larger influence on dynamic reliability while the density of mass ρ has a smaller influence on it.

a. With time increasing, the influence of variation of random parameter on dynamic reliability will also become larger.

b. With the coefficients of variation increasing, the mean value of dynamic reliability diminishes. That is to say the bigger the coefficient of variation of the random parameter, the larger the influence on the mean value of dynamic reliability of torpedo structure.

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