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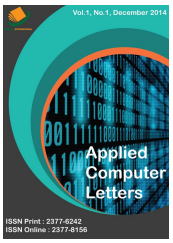

MODELING AND SIMULATION OF THE MULTI-GENERATIONS PRODUCTS DIFFUSION

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ABSTRACT

Innovative product is the key factor to business competition. A dynamic diffusion model of two generation products is established by considering the diffusion properties of multi-generational and competitive alternative effect, including expanded the application of Bass model. Most of the existing research did not consider the effect of product unit variable cost, in this model the learning effect of variable cost and product gravity are taken into consideration, and their influence to product diffusion and product profit are also analyzed. This thesis is going to describe the mathematical models of the product diffusion and will use the MATLAB to simulate the model, revealing the rationality of the model. Product diffusion is analyzed according the typical parameters' changes, after investigating model simulation results, the interactive relationship between the factors and the system structure characteristics are performed, which provide a foundation to make policies of enterprise market.

KEYWORDS

Product diffusion, multi-generations products, modeling, simulation, learning effect

1. INTRODUCTION

Based on a study, marketing diffusion means the process of a new product's being accepted by the public gradually when it goes into the market and slowly aging, and finally eliminated. Research shows that Bass model is the milestone for this field of research [1]. The product life cycle shortens with the rapid development of science and technology. A researcher analyzed the diffusion process of computer hard disk [2]. They pointed out that when the new product comes into the market, the previous one will be replaced gradually. The old product and new product have the relation of "competition" as well as "replacement". Many scholars continue to develop and refine its theoretical meaning. Taking China's communications industry as an example, a researcher introduced a multiple generation product diffusion model to display the complementary products' competition and the similar products' replacement at different generation, thus to understand the variation of the product's potential adopters, and to illustrate the effectiveness of the competitive multi- generation product diffusion model [3].

Another researcher established the multi-stage innovative diffusion model which is influenced by the price and advertisement by using the dynamic analysis, they carried out the dynamic analysis of the new product diffusion progress and also took the "Internet' as the real sample, which concluded that the diffusion system will lead to the existence of both innovative products and competitive products [4]. One of the researchers considered two competitive products which are infected, at the same time, by the new prices and advertising, as well as the influence of the solution of two homogeneous and non-homogeneous the closed- loop optimal pricing and advertising strategy of the enterprise, and pointed out that without considering the cost study effect, the optimal price stays the same, however, the optimal advertising strategy changes with time. Li Liping et al. proposed the homogeneous product diffusion model under duopoly competition, and introduced two asymmetric competitors (Mengniu and Yili) according to the brands of milk market reality in China [5,6]. They found that the optimal pricing strategy for products under the market structure of duopoly competition is mainly determined by the following factors: competition of both parties' price of the product, diffusion coefficient, rate of discount, coefficients of the attraction of the other products, and competitors' price variation trend. One of the researchers took GBM model as the research framework, putting forward the competitive diffusion model of a new product and constraint requirement of product competition for both parties [7].

Other researcher introduced free gift, product pricing strategy, and repeated purchase of merchandise into the product diffusion model and carried out dynamic analysis between the two generations' products [8]. Most of the above researches have involved in multiple stages, multiple enterprises and multiple markets. Although the way of getting profit is taken into consideration in such kind of research, the influences to the profit of the learning effects caused by the changes of the unit cost is omitted. However, product diffusion is a very complex market process. Generally it experiences multiple stages as introduction, take-off, maturity and decline. According to a study, in each stage, many factors will influence the innovation product diffusion [9]. Therefore, the interaction of the main factors in the process of new product diffusion, the complex nonlinear relationship between factors and the feasibility of the enterprise policy in the process of diffusion must be taken into account.

Therefore, on the basis of existing research this paper established the model of two generations product diffusion considering the unit cost with learning effects, and obtained the profit expression of innovative products, then analyzed the diffusion behavior of two generations' products based on the analysis of the process of product diffusion factors by using MATLAB simulation. Using dynamic simulation method to build the model to analyze the influence of each factor, this paper provided necessary basis for the decision-making for the enterprise market.

2. THE BASIC DIFFUSION MODEL

Most currently used product diffusion models are based on the Bass diffusion model, which is a two second-order system with two cumulative variables: potential customers and actual adopters. It assumes additional adopters include innovators and imitators in a period of time. Innovators refer to the group, which are affected by the advertising media to adopt the product; imitators refer to the group which accepts the product because of the communication with adopters. A group of researchers have distinguished the "innovation" from "new product" [10]. They believe that the concept of innovation has a wider range than new product does, including new products, new methods, new ideas, new system, and even the new social entities. The main research topic of innovation diffusion in current literatures is about the new product diffusion, which is what this paper concerns.

Bass model assumes that there is only one kind of product on the market, namely the product is the monopoly product; at the same time the product is durable which means the product can be continuously used for a fairly long time, the independent individual or family needs only one product. The amount of products and sales is equivalent to the number of consumers who have adopted products. According to this model is as follows:

$$f(t)/[1-F(t)] = p+qf(t) \qquad (1)$$

Where f(t) is the time density function of adopters at time t, F(t) is ratio of cumulative adopters to maximum adopters (i.e. market scale). The innovation coefficient and imitation coefficient are denoted by p and q respectively. Suppose m is the total market scale, N(t) is the cumulative adopters, n(t) is the adopters, as shown in Equations (2) and (3):

$$n(t) = mf(t);\ N(t) = mF(t) \qquad (2)$$

$$n(t)=dN(t)/dt = p[m - N(t)] + q/m \cdot N(t)[m - N(t)] \qquad (3)$$

Equation (3) can be divided into two parts: The first part "p[m-N(t)]" represents the number of adopters decide to buy new products influenced by external factors; The second part "q/m·N(t)[m-N(t)]" expresses the customers decide to adopt the products affected by the communication between previous buyers and the potential customers. To solve the model, we can obtain an analytical solution expressed as Equations (4) and (5) [11].

$$N(t) = m(1-e^{-(p+q)t})/(q/p \cdot e^{-(p+q)t}+1) \qquad (4)$$

$$n(t) = m((p+q)^2/p)\cdot\left[e^{-(p+q)t}/(q/pe^{-(p+q)t}+1)^2\right] \qquad (5)$$

Existing research shows that innovation coefficient p varies between 0.0007 ~ 0.03, imitation coefficient q is between 0.38~0.53, which shows the internal diffusion process is driven more by oral communication of information, but the importance of the public media information cannot be ignored [12]. Then many researchers have extended the application of Bass model, such as consideration of price level at which consumers expect, the influence of market scale.

3.THE BASIC DIFFUSION MODEL MULTI-GENERATION PRODUCTS DIFFUSION MODELS AND THEIR ANALYSIS

The basic diffusion model have been analysed comprehensively, combined with the diffusion of multi-generation products, making the following assumptions for this model: (1) Innovative products do not change itself throughout its life cycle; (2) The geographical restrictions of the social system do not change with the diffusion process, the situation like the closed-door policy does not appear, so the potential market is consistent; (3) The diffusion process is divided into two stages: non-adoption and adoption; (4) The two generations' products belong to the same enterprise, and diffusion process has no supply constraints; (5) The second generation product is an upgraded version of the first generation product; (6)

Each consumer only buys one generation product; consumers will not buy the first generation products if they have already bought the second generation. In the problem of new product diffusion, the interactions between products are reflected by the preferences of customers to different products. When only the first generation products exist in the market, the diffusion model and Bass model of the product are described as the same form. While new products appear in the market, the market structure will be changed.

On the one hand, compared to the first generation of products, the new product's function will be somewhat upgraded, so some new consumers will emerge into the market and decide to purchase the new product; on the other hand, some consumers who have prepared to buy the first generation of product originally, may be attracted by the new products. This feature is reflected in the model through that previous generation product sales will be less than the original sales and new product market potential is expanded. Description of mathematical symbols in the model:

Table 1: Symbols and Notations.

Symbols	meanings
T	innovative product sales cycle
t_0	Second generation product market entry time
a_i	innovation coefficient of i generation product
b_i	imitation coefficient of i generation product
$x_i(t)$	cumulative adopters of i generation product
r	the discount rate
F_i	i generation product cost for entering the market
m_i	i generation product market potential
m	total market /m=m1+m2
$P_i(t)$	i generation product price
σ	product gravity
$U(t)$	control variable; the value of which is 0 before the entry of second generation , and 1 after then
UVC_i	unit variable cost of i generation product
π_i	i generation product profit

Definition. Product gravity σ is the ability for the product to occupy the market share, namely the influence coefficient of new product to the previous generation product. We believe that the second generation is negative correlated with the price, that is, low price products will occupy more market share, and therefore we have the following definition (6) Definition. Product gravity σ is the ability for the product to occupy the market share, namely the influence coefficient of new product to the previous generation product. We believe that the second generation is negative correlated with the price, that is, low price products will occupy more market share, and therefore we have the following definition (6)

$$\sigma = 2P_1(t)/(P_1(t)+P_2(t)) \qquad (6)$$

3.1 Multi-generation Diffusion Model

A new generation products will not immediately replace the previous generation ones, but to compete with them. When there are only one generation products in the market, assuming that diffusions of products satisfy Bass form. Then we obtain the following model:

$$\frac{dx_1(t)}{dt} = a_1\left[m_1 - x_1(t)\right] + \frac{b_1}{m_1}x_1(t)\left[m_1 - x_1(t)\right] \qquad (7)$$

When new products appear in the market, the market structure will be changed. This can be reflected in the model as previous generation product sales decline while new product market potential rises.

Suppose that a little amount of second generation products will be given to the customers freely at the initial time, thus x_2 has an initial value of x_{20}, and then we obtain the following models.

$$\frac{dx_1(t)}{dt} = \left[a_1 + \frac{b_1}{m_1}x_1(t)\right]\left[m_1 - x_1(t)\right] - \frac{\sigma}{m}u(t)x_2(t)\left[m_1 - x_1(t)\right] \qquad (8)$$

$$\frac{dx_2(t)}{dt} = \left[a_2 + \frac{b_2}{m_{2+}\frac{\sigma}{m}u(t)x_2(t)\left[m_1 - x_1(t)\right]}x_2(t)\right]\left[m_2 + \frac{\sigma}{m}u(t)x_2(t)\left[m_1 - x_1(t)\right]\right]u(t) - x_2(t) \qquad (9)$$

To facilitate the analysis of the diffusion model, the following contents are introduced.

The diffusion equations express products' competition and replacement. After entering the market, the second generation adopters denoted by f2 will reduce the potential adopters of the first generation products .The reductive quantity is denoted

$$\text{as} \frac{\sigma}{m} u(t)x_2(t)[m_1 - x_1(t)] .$$

For company, each generation products aim to get as much profit as possible. Therefore, we consider the time value of capital and each profit discount, let $dx_1(t)/dt = f_1$,$dx_2(t)/dt = f_2$, then we obtain the following mode for decision targets (10).

$$\max\pi=\pi_1+\pi_2$$

$$\text{s.t} \quad \max_{P_1} \pi_1 = \int_0^T e^{-rt}[(P_1(t)-\mathrm{UVC}_1)f_1]\mathrm{d}t - F_1$$

$$\max_{P_2,u} \pi_2 = \int_t^T e^{-rt}\left[(P_2(t)-\mathrm{UVC}_2)f_2\right]\mathrm{d}t - F_2e^{-rt} \quad (10)$$

$$f_1 = \begin{cases} model(7) & t \le t_0, T > t_0 \\ model(8) & t > t_0, \ T > t_0 \end{cases}$$

However, the average cost in the long time with production, design and management experience improved will have a decreasing trend [13]. We call it "learning effect", which can be displayed as a "learning curve". Learning effect of product's unit variable cost is expressed as Equation (11)

$$\mathrm{UVC}_i == \mathrm{UVC}(1)\cdot x_i(t)^{-\theta} \qquad (11)$$

UVC(1)--init unit variable θ --the coefficient of elasticity with production changes.

According to the learning curve theory, learning rate refers ratio to which when the products throughput doubled, the unit cost or production time is decreased [13], which is denoted by δ. Definition can be simplified into (12):

$$\begin{aligned} \delta &= \mathrm{UVC}(2x_i(t))/\mathrm{UVC}(x_i(t)) \\ &= \left[\mathrm{UVC}(1)(2x_i(t))^{-\theta}\right]/\left[\mathrm{UVC}(1)(x_i(t))^{-\theta}\right] \quad (12) \\ &= 2^{-\theta} \end{aligned}$$

According to different production tasks, learning rate is also different, usually is between 70%-95%. The more complex the task, learning effect is more obvious, and the rate of learning is lower. Thus, we can get the $\theta = -\log_2\delta$, showing that it generally ranges from 0.07 to 0.5.

3.2 Multi-generation Diffusion Model Analysis

Take profit maximization in a certain sales period as the objective. It is necessary to make decisions about the market-entry time of second generation products due to the uncertain demand and circumstances. Proposition. The best entry time for the second generation of products $t_0 = -[\ln(m_1a_1 - M_0a_1)/(M_0b_1 + m_1a_1)]/(a_1 + b_1)$ Proof. According to the above description, accumulative profit can be expressed as (13)

$$\begin{aligned} \frac{\mathrm{d}\pi}{dt} &= f_1(P_1(t) - UVC_1)e^{-rt} - f_1(P_1(t) - UVC_1)e^{-rt} \\ &\quad - f_2(P_2(t) - UVC_2)e^{-rt} + rF_2e^{-rt} \\ &= -f_2(P_2(t) - UVC_1)e^{-rt} + rF_2e^{-rt} \\ &= -[(a_2 + \frac{b_2}{m_2}x_{20})(m_2 + \frac{\sigma}{m}x_{20}(m - x_1(t)) - x_{20}) \\ &\quad (P_2(t) - UVC_1)e^{-rt} + rF_2e^{-rt} \end{aligned} \qquad (13)$$

If $\frac{\mathrm{d}\pi}{dt} = 0$ holds, model can be simplified into

$$x_1(t) = m_1 - [rF_2/((P_2(t) - \mathrm{UVC}(1)\cdot x_{20}^{-\theta}) (a_2 + \frac{b_2}{m}x_{20})) + x_{20} - m_2]/\sigma x_{20}$$

Suppose $x_1(t) = M_0$, according to the model (4), we can get $M_0 = m_1(1 - e^{-(a_1+b_1)t})/(b_1/a_1 \cdot e^{-(a_1+b_1)t} + 1)$, then we obtain the best entry time of the second generation products:

$t_0 = -[\ln(m_1a_1 - M_0a_1)/(M_0b_1 + m_1a_1)]/(a_1 + b_1)$

4. SIMULATION ANALYSIS

Model (10) is a nonlinear system. The decision variables are the amount of two generations of products. Therefore, this section mainly focuses on the analysis of the basic numerical model simulation. The paper analyzes the influences of models of diffusion process in different scenarios on the two generation products. The sensitivity analysis is performed to see how the typical parameters would affect the variables in the model. The parameters of model are a_1 =0.005, b_1 =0.45, a_2 =0.006, b_2 =0.55, m_1 =500000, m_2 =1000000, t_0=6, T=40, σ=1[14]. For convenience sake, assuming that the two generation product is developed by the same company.

4.1 The Basic Simulation Analysis

Suppose that there are two generation of innovative products in the market, and then the model is simulated by MATLAB, the simulation results are shown in Figures 1 and 2.

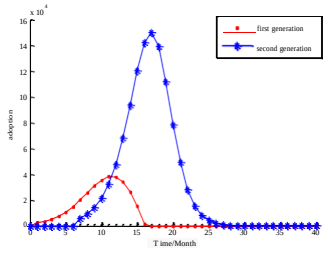


Figure 1: Multi-generation diffusion of adopters.

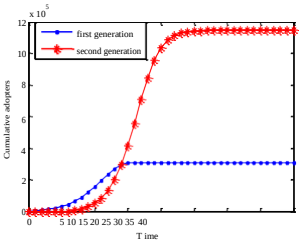


Figure 2: Multi-generation diffusion of cumulative adopters.

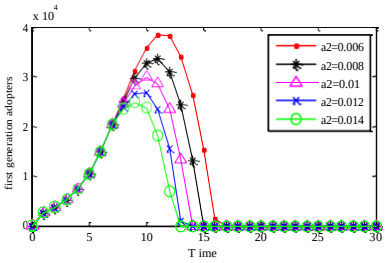
From the simulation results, the following conclusions can be drawn: Products of the first generation and the second generation diffuse in the market, shown as an "S" curve, which is similar to Bass model. As the two generation products enter into the market at different times, the diffusion curve radians, peak values and other aspects of the two are not identical. The first generation of products to enter the market is early; the diffusion adopters increased steadily in a few periods and reached a peak sale 40000 in the 11th period, then the first generation products fade out from the market until it is eliminated at 16th period. Compared with the first generation of products, second products' diffusion process is also different. It is more competitive than the first generation products. The second generation products are adopted by more and more consumers, the amount of which reaches 150000. Finally, the sale of second generation products is close to 0 at the 26th period. Explanations for the diffusion modes mainly include the following two points: First, the amount of the second generation products market potential is greater than the former generation ones. Therefore, when it enters the market, it will attract more potential consumers, so the sales also will be larger; Second, the second generation products have an influence on the first generation products after its entry to the market. Part of the customers will change their minds to purchase a new generation products, which further expands the second-generation market capacity and speeds up its spread.

4.2 Sensitivity Analysis of Innovation and Imitation Coefficient

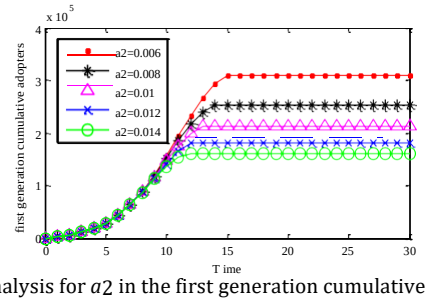
Hereafter we will change the basic conditions of the market. We will observe how the parameters will be changed with the variation of innovation coefficient and imitation coefficient.

Set a_1, b_1, b_2 to be constant. Innovation coefficient a_2 is changed from 0.006 to 0.014; we can get changes of two different generation products for the current adopters and cumulative adopters. We can find that innovation coefficient significantly affects the diffusion results of two generations products. The simulation results are shown as Figures 3(a), (b), (c), and (d). With the increase of innovation coefficient, the second generation product adopters and the cumulative adopters also increase, but not significantly; while the first generation product adopters and the cumulative adopters are significantly reduced. The second generation products diffusion inhibits the diffusion of the first generation products, which reduces peak point of the first generation products, and shortens the first generation product cycle.

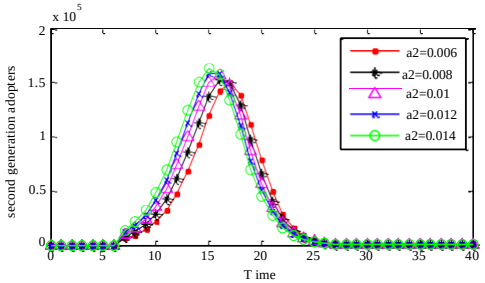
Moreover, let us analyze the model's sensitivity to the imitation coefficient. Set a1, b1, a2 to be constant. Imitation coefficient b2 is changed from 0.45 to 0.65. The simulation conclusions is shown as Figures 3(e), (f), (g), and (h), through which we can draw the conclusions that: With the increase of the second generation product imitation coefficient, the second generation product adopters and the cumulative adopters will significantly increase, while the first generation product adopters and the cumulative adopters is reduced. The second generation products diffusion inhibits the diffusion of the first generation, which shortens the first generation cycle. We can also conclude that innovation coefficient's increase plays a greater role in shortening the former generation product life cycle and reducing its peak point; The increase of imitation coefficient will more greatly raise this generation product peak point.



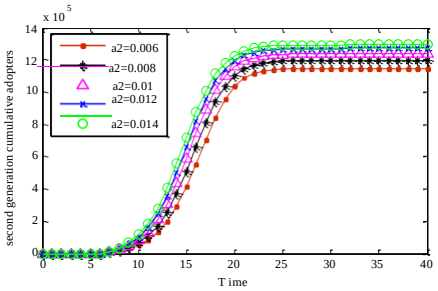
(a) Sensitivity analysis for $a2$ in the first generation adopters.



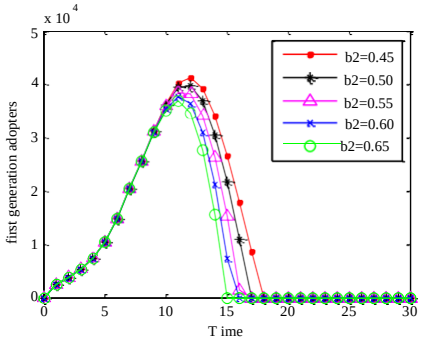
(b) Sensitivity analysis for $a2$ in the first generation cumulative adopters.



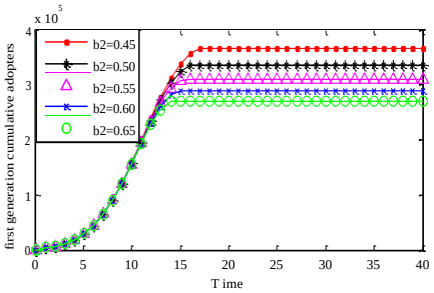
(c) Sensitivity analysis for $a2$ in the second generation adopters.



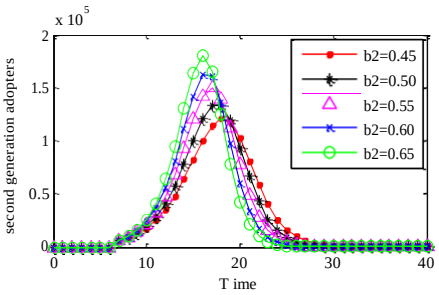
(d) Sensitivity analysis for $a2$ in second generation cumulative adopters.



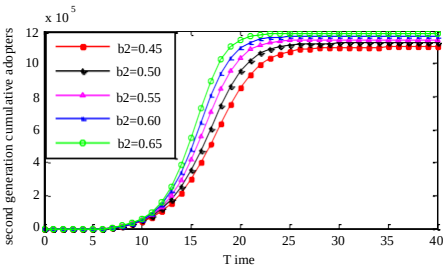
(e) Sensitivity analysis for $b2$ in the first generation adopters.



(f) Sensitivity analysis for $b2$ in the first generation cumulative adopters.



(g) Sensitivity analysis for $b2$ in the second generation adopters.



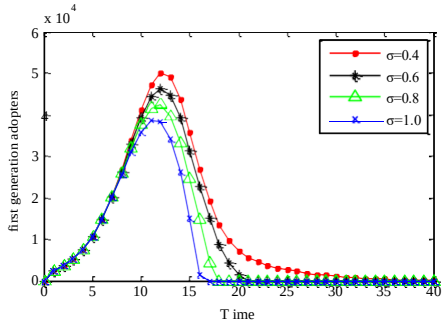
(h) Sensitivity analysis for b2 in second generation cumulative adopters.

Figure 3: The sensitivity analysis of market parameters.

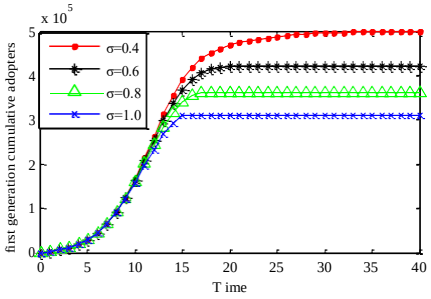
4.3 The Sensitivity Analysis of Product Gravity "σ"

According to Definition 1, influence coefficient of the second generation product to the first generation product mainly depends on two generations product price. Assuming that the second generation product is upgraded from the first generation product, then its function will be more perfect than the first generation product, so product gravity is $0<\sigma \le 1$.

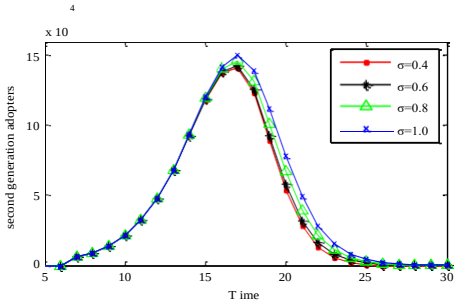
In order to examining the competitive relationship between the two generations product, we will keep other parameters fixed and increase σ value from 0.4 to 1. Simulation results are shown in Figures 4(a), (b), (c), and (d). As influence coefficient of the second generation product to the first generation product increases, namely the price of the second generation product gradually reduces, the first generation adopters and cumulative adopters are reduced, while the amount of second generation product adopters and cumulative adopters are rising, suggesting that changes in the price has a significant impact on product diffusion.



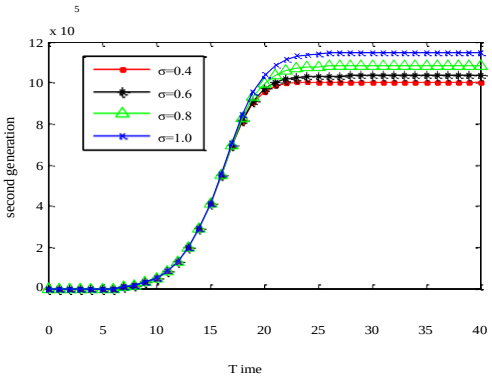
(a) Sensitivity analysis for σ in the first generation adopters.



(b) Sensitivity analysis for σ in the first generation cumulative adopters.



(c) Sensitivity analysis for σ in the second generation adopters.



(d) Sensitivity analysis for σ in the second generation cumulative adopters.

Figure 4: Sensitivity analysis of the gravity to the model.

5.THE INFLUENCE OF LEARNING EFFECTS TO PRODUCT PROFIT ANALYSIS

According to the learning curve theory, as workers have more and more experience, the unit variable costs could be shown a trend of decrease. Keeping other parameters constant, learning effect of products increases from 0.1 to 0.18. Simulation results are shown in Figure 5. When the learning effect is greater, the enterprise can obtain more profits. Learning effects affect profits. Companies which are developing a new generation product need to draw experiences and lessons from the previous generation of products, constantly improve the production experience, to reduce the variable costs of the product.

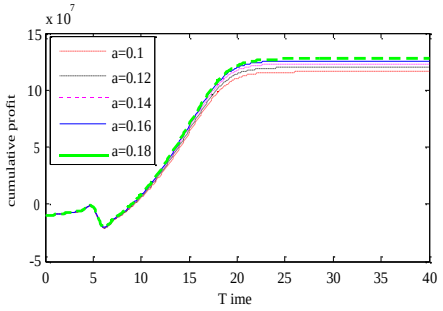


Figure 5: The influence analysis of learning effects to product profit

6. SIMULATION ANALYSIS

This article has considered the learning effects into the competitive products. We discovered that learning effect has a direct influence on the product profit. Simulation results show that by setting reasonable parameters and the system structure, the system model could have a certain practical significance which can simulate the real competitive product diffusion process. This provides a useful auxiliary tool for product market development decisions.

For simplicity, this paper assumes that the spread of the two products conform to Bass model, on the basis of which to study how the market parameters and two generations products' influence coefficient would affect system behavior, and does not give a theoretical conclusion of how the price change would influence product diffusion. Moreover, this paper only described the products' competition and substitution effect within a single enterprise, with no consideration to products from different enterprises. This will provide direction for our future research.

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